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The Effects of Differentiated Emission Taxes

by

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Abstract

Extending the standard 2×2 Heckscher-Ohlin-Samuelson model to include emission as the third production factor, we consider the effects of emission taxes on outputs and the level of emission. The following results are derived. First, an increase in emission tax imposed on one industry may paradoxically increase the outputs of the industry. Second, an increase in emission tax imposed on one industry may raise the total level of emission.

Keywords: Emission; the Heckscher-Ohlin-Samuelson model; differentiated emission taxes.

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1 Introduction

Environmental problems such as air and water pollution have been serious problems in industrial countries for a long time. In recent years, the global warming caused by the accumulation of green-house gas such as carbon dioxide has been recognized as a new big problem in the world. Regional problems such as wastes disposal are also increasing in many urban areas. All of these problems have a common feature, that is, one of the causes of these problems is by-products from various productive activities (we call such by-products *emissions* in this paper).

With the emergence of these problems, the regulations on emissions are recognized as important policy subjects. Recently, there is a tendency that, in stead of the traditional direct regulations, the regulations by tradable emission permits have been attracted the more and more attention from both economists and policy makers. For example, the Kyoto Protocol in 1997 includes a plan to introduce international permit trades for green house gas among developed countries in the future, and several countries have already introduced some kinds of permit trades in the domestic markets.

Why do permit trades attract much attention? The main reason is that if emission permit trades work well, they can not only regulate the level of emission, but realize economic efficiency in the sense that costs of emission abatement are minimized.¹ However, it is also pointed out that there are several impediments for permit trades to operate well.² Moreover, as to permit trades for green house gas cited above, there are sharp conflicts on trading rules among the governments of participant countries.

On the other hand, there is another method for regulation which is also market-oriented but more traditional one, that is, taxation on emissions. Various kinds of emission taxes have already been introduced in several European countries. For example, carbon taxes in Scandinavian countries, taxes on electricity and gasoline in Germany, etc. What is worth pointing out is that to implement emission tax, some of the problems attaching to permit trade do not arise. Especially, to make permit trade work well, it is necessary to make a competitive permit market. This problem does not arise in implementing emission tax. Moreover, in contrast to permit trade, a lot of knowledge about regulations by tax has already been accumulated. In this sense, it is said that emission tax is an easier way for regulation.

¹Moreover, it is believed that market oriented regulations like permit trades require much less informational and administrative costs than direct regulations.

²Sartzetakis and McFetridge (1999) p.47 enumerate several impediments for permit trades: (i) small numbers of participants, (ii) transaction costs, (iii) cheating and monitoring costs, (iv) costs of transition from pre-existing regulations, (v) uncertainty that leads to the hoarding of permits.

As to formal economic analysis, there are already many theoretical works which analyze emission regulations or an economy with emissions. In general equilibrium models, emissions are usually introduced into the models as production factors although emissions are, in reality, joint products of outputs. For example, Pethig (1976), Yohe (1979), Siebert, Eichverger, Gronych and Pethig (1980), Hertel (1988), Copeland and Taylor (1994, 1995), and Ishikawa and Kiyono (2000).

All of these analyses presume the situations that all industries in a country are regulated equally. In the case of emission tax, they assume the uniform tax rate or the uniform change in emission tax across industries. However, there are many cases in which these presumptions are not appropriate. There are at least two reasons for this. First, emission regulations often receive strong oppositions from industries, especially from those who are greatly affected by the regulations. Faced with these strong oppositions, the regulators may be forced to implement regulations in non-uniform ways. Second, even if the regulations are imposed uniformly on all industries, it may be that some kinds of subsidiary policies are simultaneously implemented on some industries. In this case, the strength of regulations comes to vary across industries. As an example, there is the case of Sweden: when Sweden introduced carbon taxes in 1991, some industries were exempted from taxes. (see OECD, 1994)

For these reasons, this paper attempts to analyze the situation where emission taxes are not imposed uniformly across industries. Hoel (1996) has already dealt with a similar issue. He considered the situation where there are both participants and non-participants to an international environmental agreement and derived the optimal emission tax formula for participating countries. However, he did not analyze in detail the effects of emission tax on outputs and the level of emission.

We employ the standard 2×2 Heckscher-Ohlin-Samuelson (HOS) model, and, as in the literature, we treat emission as the third production factor.³ Thus, the model has the similar structure as the 2×3 HOS model. Let us explain the reason to consider two primary factors. First, it becomes the extension of the basic 2×2 model. Second, we can consider the difference between the degree of substitution between capital and emission and that between labor and emission which is neglected in one primary factor model. The final, and most critical reason is that, as shown in Ishikawa and Kiyono (2000), Section 3, when emission tax is imposed in a two-sector and one-factor small open economy, incomplete specialization equilibrium is unstable and thus, complete specialization always realizes. As mentioned above, we intend to explore the inter-industry difference in the

³For details of this approach, see, for example, Pethig (1976) or Copeland and Taylor (1994)

strength of regulations. However, if production is specialized, this difference makes no sense. That is why we consider two primary factors.

We focus on the production side of the economy. Thus, we assume that both commodity prices and factor endowments are given exogenously.⁴ We analyze the effects of emission taxes on outputs and the level of emission. The following results are obtained. First, strengthening emission tax imposed on an industry may increase the outputs of the industry. Second, while strengthening emission taxes uniformly across industries always reduces emissions, strengthening emission tax unevenly may increase them.

The ‘carbon-leakage’ problem is frequently argued.⁵, and The ‘carbon-leakage’ refers to the following phenomenon: When some countries unilaterally reinforce emission regulations, the activities generating emissions shift to unregulated countries through the change in the trade pattern, movements of firms, or the fall in the prices of fossil fuel etc., and as a result, the effectiveness of regulations is weakened or, in the worst case, completely canceled out. The second result of this paper means that the phenomenon like international carbon-leakage can occur among industries within a country.

2 The Model

We consider a two-sector two-factor economy. However, we treat emission as the third production factor although emission is a by-product from production activity. Therefore, the model has the similar structure to the standard 2×3 HOS model employed in Batra and Casas (1976) and Jones and Easton (1983). However, there is one important difference from their model, that is, while all factor prices are endogenously determined in the standard 2×3 model, the factor prices corresponding to emission in our model are policy instruments determined exogenously. The model more similar to ours is the 2×3 model with capital mobility like Wong (1995), chapter 4 because the one of the factor prices in his model (the rental rate) is also constant.

Let v_j^i denote the amount of factor j ($j = K, L$) employed in sector i ($i = 1, 2$) and v_Z^i denote the level of emission from sector i . The production function of sector i is given by $Q_i = f^i(v^i)$ where Q_i is the output of sector i and $v^i = (v_K^i, v_L^i, v_Z^i)$. We assume that $f^i(\cdot)$ is concave and homogeneous of degree one in v^i . By linear homogeneity, cost-minimizing input coefficient is independent of output.

⁴Or we can treat the model as a small open economy.

⁵For example, see Ishikawa and Kiyono (2000) and Ulph (1993)

The cost function of sector i is

$$c^i(w_K, w_L, w_Z^i) \equiv \min_{\{a_{Ki}, a_{Li}, a_{Zi}\}} \{w_K a_{Ki} + w_L a_{Li} + w_Z^i a_{Zi} \mid f^i(a_{Ki}, a_{Li}, a_{Zi}) \geq 1\} \quad (1)$$

where w_j ($j = K, L$) denotes the price of factor j and w_Z^i denotes the specific emission tax imposed on sector i . From Shephard's lemma, $a_{ji}(w_K, w_L, w_Z^i) = \partial c^i(w_K, w_L, w_Z^i) / \partial w_j$.

At a competitive equilibrium, the unit cost must be equal to the price if the commodity is actually produced. Thus,

$$c^i(w_K, w_L, w_Z^i) = p_i \quad i = 1, 2 \quad (2)$$

As to capital and labor, endowments are given exogenously as in the standard 2×2 model, thus full employment requires

$$\sum_{i=1,2} a_{ji} Q_i = v_j \quad j = K, L \quad (3)$$

where v_j is the endowment of factor j ($j = K, L$). Given commodity prices, factor endowments, and emission taxes, equilibrium factor prices and outputs are determined by (2) and (3). The level of emission from sector i is given by

$$v_Z^i = a_{Zi} Q_i \quad i = 1, 2 \quad (4)$$

To do comparative statics, we need to relate changes in endogenous variable with changes in exogenous variables. Let θ_{ji} be the cost share of factor j in sector i and λ_{ji} the fraction of factor j employed in sector i . Then, equations of change are given by

$$\sum_{j=K,L} \theta_{ji} \hat{w}_j = \hat{p}_i - \theta_{Zi} \hat{w}_Z^i \quad i = 1, 2 \quad (5)$$

$$\sum_{i=1,2} \lambda_{ji} \hat{Q}_i = \hat{v}_j - \sum_{i=1,2} \lambda_{ji} \hat{a}_{ji} \quad j = K, L \quad (6)$$

where a hat over a variable denotes the rate of change (e.g. $\hat{w}_j = dw_j/w_j$).

We define, for $\forall i, h = 1, 2 \ i \neq h, \forall j, l = K, L, Z \ j \neq l$

$$|\theta_{jl}^{ih}| \equiv \theta_{ji} \theta_{lh} - \theta_{li} \theta_{jh} \quad |\lambda_{jl}^{ih}| \equiv \lambda_{ji} \lambda_{lh} - \lambda_{li} \lambda_{jh} \quad (7)$$

For simplicity, we assume that $w_Z^1 = w_Z^2$ at an initial equilibrium, that is, the emission taxes on both industries are set at the identical level at an initial equilibrium. Under this assumption, it can easily be shown that if $a_{j1}/a_{j2} > a_{l1}/a_{l2}$, we have $|\theta_{jl}^{12}| > 0$ and $|\lambda_{jl}^{12}| > 0$. In other words, the signs of $|\theta_{jl}^{12}|$ and $|\lambda_{jl}^{12}|$ depend only on the factor intensity between factor j and l .

From (5) and (6), the following relations are derived.

$$\hat{w}_K = \frac{1}{|\theta_{KL}^{12}|} [\theta_{L2}(\hat{p}_1 - \theta_{Z1}\hat{w}_Z^1) - \theta_{L1}(\hat{p}_2 - \theta_{Z2}\hat{w}_Z^2)] \quad (8)$$

$$\hat{w}_L = \frac{1}{|\theta_{KL}^{12}|} [-\theta_{K2}(\hat{p}_1 - \theta_{Z1}\hat{w}_Z^1) + \theta_{K1}(\hat{p}_2 - \theta_{Z2}\hat{w}_Z^2)] \quad (9)$$

$$\hat{Q}_1 = \frac{1}{|\lambda_{KL}^{12}|} (\lambda_{L2}\hat{v}_K - \lambda_{K2}\hat{v}_L + \lambda_{L2}\hat{\beta}_K - \lambda_{K2}\hat{\beta}_L) \quad (10)$$

$$\hat{Q}_2 = \frac{1}{|\lambda_{KL}^{12}|} (\lambda_{K1}\hat{v}_L - \lambda_{L1}\hat{v}_K + \lambda_{K1}\hat{\beta}_L - \lambda_{L1}\hat{\beta}_K) \quad (11)$$

where $\hat{\beta}_j \equiv -(\lambda_{j1}\hat{a}_{j1} + \lambda_{j2}\hat{a}_{j2})$.

In the standard 2×3 model, factor prices usually depend not only on commodity prices, but on factor endowments, therefore, the Stolper-Samuelson and Rybczynski do not hold in the same way as the standard 2×2 model and they are at most valid in the weak sense (see Batra and Casas, 1976, theorem 6 and theorem 9).⁶

However, as Wong (1995) chapter 4 shows, although the model we consider here has three factors, both theorems are valid like the 2×2 model. The intuitive reason for this is very simple. Although there are three factors, the factor price corresponding to emission is the emission tax which is exogenously given constant. Therefore, like in the 2×2 model, the factor prices (w_K and w_L) only depend upon commodity prices and do not depend upon factor endowments, and it follows that this model has the same structure as the 2×2 model. Therefore we can treat the model in the same way as the standard 2×2 model as to the relation between commodity prices and factor prices and the relation between factor endowments and outputs.

3 The effects of emission tax on outputs.

In the remainder of the paper, we focus on the effects of the change in the emission taxes. Thus, we set $\hat{p}_i = \hat{v}_j = 0$ for $\forall i = 1, 2$ and $j = K, L$.

First, let us examine the effects on outputs. We define some notations.

$$\varepsilon_{jl}^i \equiv \frac{w_l}{a_{ji}} \frac{\partial a_{ji}}{\partial w_l} \quad i = 1, 2 \quad j, l = K, L, Z$$

That is, ε_{jl}^i is the price elasticity of factor demand per output in sector i . If $\varepsilon_{jl}^i > (<) 0$,

⁶However, as Suzuki (1983) pointed out, the theorem 6 and 9 in Batra and Casas (1976) include some errors.

factor j and l are called substitutes (complements) in sector i .⁷ ε_{jl}^i has the following properties:

1. $\varepsilon_{lj}^i = \theta_{ji}\varepsilon_{jl}^i/\theta_{li}$. Note that ε_{jl}^i is not symmetric (i.e. $\varepsilon_{jl}^i \neq \varepsilon_{lj}^i$).
2. By the zero homogeneity of a_{ji} with respect to (w_K, w_L, w_Z^i) , for $\forall j = K, L, Z$

$$\varepsilon_{jK}^i + \varepsilon_{jL}^i + \varepsilon_{jZ}^i = 0 \quad (12)$$

3. By the concavity of cost function, $\varepsilon_{jj}^i \leq 0$ and $\varepsilon_{jj}^i \varepsilon_{ll}^i - \varepsilon_{jl}^i \varepsilon_{lj}^i \geq 0$.

From these properties, if ε_{jl}^i is negative, both ε_{jk}^i and ε_{lk}^i must be positive, that is, there is at most one pair of factors which are complements. Moreover, property 2 and 3 imply that the following inequality holds for $\forall i = 1, 2, \forall j, l, k = K, L, Z, j \neq l, l \neq k, k \neq j$:

$$\varepsilon_{jl}^i \geq -\frac{\theta_{li}\varepsilon_{jk}^i\varepsilon_{lk}^i}{\theta_{ji}\varepsilon_{jk}^i + \theta_{li}\varepsilon_{lk}^i} \quad (13)$$

This means that even if $\varepsilon_{jl}^i < 0$ (i.e. factor j and l are complements), the degree of complementarity is limited by some bound.

By the definition of ε_{jl}^i , $\hat{a}_{ji} = \sum_{l=K,L,Z} \varepsilon_{jl}^i \hat{w}_l$. Thus, we have

$$\lambda_{j1}\hat{a}_{j1} + \lambda_{j2}\hat{a}_{j2} = \sum_{l=K,L} (\lambda_{j1}\varepsilon_{jl}^1 + \lambda_{j2}\varepsilon_{jl}^2)\hat{w}_l + \lambda_{j1}\varepsilon_{jZ}^1\hat{w}_Z^1 + \lambda_{j2}\varepsilon_{jZ}^2\hat{w}_Z^2$$

In addition, we define $\varepsilon_{jl} \equiv \lambda_{j1}\varepsilon_{jl}^1 + \lambda_{j2}\varepsilon_{jl}^2$. ε_{jl} expresses the price elasticity of total factor demand and has the similar properties to ε_{jl}^i : (i) $\varepsilon_{lj} = \alpha_j \varepsilon_{lj}/\alpha_l$ where α_j is the share of factor j in national income ($\alpha_j = w_j v_j / Y, Y = \sum_{i=1,2} p_i Q_i$). (ii) $\varepsilon_{jK} + \varepsilon_{jL} + \varepsilon_{jZ} = 0$ for $\forall j = K, L, Z$. (iii) $\varepsilon_{jj} \leq 0$.

Using above notations, we can rewrite $\hat{\beta}_j$ as follows

$$-\hat{\beta}_j = \varepsilon_{jK}\hat{w}_K + \varepsilon_{jL}\hat{w}_L + \lambda_{j1}\varepsilon_{jZ}^1\hat{w}_Z^1 + \lambda_{j2}\varepsilon_{jZ}^2\hat{w}_Z^2 \quad (14)$$

Combining (8)-(11), and (14), we can derive the expression of \hat{Q}_i : for $\forall i, h = 1, 2, i \neq h$

$$\hat{Q}_i = \frac{1}{|\theta_{KL}^{ih}||\lambda_{KL}^{ih}|} (A_i^i \hat{w}_Z^i + A_h^i \hat{w}_Z^h) \quad (15)$$

⁷ $\varepsilon_{jl}^i/\theta_{li}$ is the famous Allen's partial elasticity of substitution (see Chambers, 1988, p. 95). While most papers including Batra and Casas (1976), Yohe (1979), and Siebert et al. (1980) use this Allen's measure of elasticity, we use ε_{jl}^i as Jones and Easton (1983).

where

$$\begin{aligned}
A_i^i &= -B_i^i \theta_{Zi} + |\theta_{KL}^{ih}| (\lambda_{Kh} \lambda_{Li} \varepsilon_{LZ}^i - \lambda_{Lh} \lambda_{Ki} \varepsilon_{KZ}^i) \\
A_h^i &= -B_h^i \theta_{Zh} + |\theta_{KL}^{ih}| (\lambda_{Kh} \lambda_{Lh} \varepsilon_{LZ}^h - \lambda_{Lh} \lambda_{Kh} \varepsilon_{KZ}^h) \\
B_i^i &= \frac{\alpha^h}{\alpha_L} (\theta_{Kh} + \theta_{Lh})^2 \varepsilon_{KL} + \frac{\alpha^h}{\alpha_L} \theta_{Lh}^2 \varepsilon_{KZ} + \frac{\alpha^h}{\alpha_K} \theta_{Kh}^2 \varepsilon_{LZ} \\
B_h^i &= -\frac{\alpha^h}{\alpha_L} (\theta_{Kh} + \theta_{Lh}) (\theta_{Ki} + \theta_{Li}) \varepsilon_{KL} - \frac{\alpha^h}{\alpha_L} \theta_{Lh} \theta_{Li} \varepsilon_{KZ} - \frac{\alpha^h}{\alpha_K} \theta_{Kh} \theta_{Ki} \varepsilon_{LZ} \\
\alpha^i &= p_i Q_i / (p_1 Q_1 + p_2 Q_2)
\end{aligned}$$

Since both $|\theta_{KL}^{ih}|$ and $|\lambda_{KL}^{ih}|$ have the same signs, the fraction $1/|\theta_{KL}^{ih}| |\lambda_{KL}^{ih}|$ is always positive. Note that the signs and size of all terms depend not only on factor intensities (i.e. θ_{ji} and λ_{ji}), but also on the elasticities of substitution between factors (i.e. ε_{jl}^i).

Now, we can show the following paradoxical proposition.

Proposition 1 *The sign of \hat{Q}_i/\hat{w}_Z^i may be positive, that is, strengthening emission tax imposed on an industry may increase the outputs of the industry.*

We show this proposition by giving one numerical example. Suppose that $\theta_{Ki} = \theta_{Lh} = 0.1$, $\theta_{Li} = \theta_{Kh} = 0.8$, $\theta_{Zi} = \theta_{Zh} = 0.1$, $\varepsilon_{KL}^i = \varepsilon_{KZ}^i = 1$, $\varepsilon_{LZ}^i = -0.06$ and that in sector h , capital and labor are perfect complements (i.e. $\varepsilon_{Kj}^h = \varepsilon_{Lj}^h$ for $\forall j = K, L, Z$). In this case, $\hat{Q}_i/\hat{w}_Z^i > 0$ (see Appendix 1).

The proposition is very counter-intuitive because the rise in emission tax should have the cost-push effect and thus, lead to the downward pressure on the output of the industry. However, a close look at A_i^i in (15) reveals that, in addition to the cost-push effect, the rise in emission tax has another effect.

Two effects can be explained as follows. First, the sector specific rise in emission tax alters factor prices in the same way as the fall in the commodity price (see the RHS of (5)). These changes in factor prices lead to the changes in factor demand and the outputs are adjusted so as to clear factor markets. This effect, which we call the cost-push or indirect effect, is represented by the first term in A_i^i . In addition, the change in emission tax directly affects factor demand through substitution (or complement) between factors. This substitution or direct effect is represented by the second term.

For example, suppose that w_Z^i rises by one percent. This has the same impact on factor prices as θ_{Zi} percent fall of p_i , and its impact on Q_i is represented by $-B_i^i \theta_{Zi}$. We can show that this indirect effect of the rise in w_Z^i through factor price adjustment always decreases the output, i.e. $B_i^i \geq 0$, (see Appendix 2).

On the other hand, one percent rise in w_Z^i raises the demands for capital and labor by $\lambda_{Ki}\varepsilon_{KZ}^i$ and $\lambda_{Li}\varepsilon_{LZ}^i$ respectively (or reduces them if they are complements). If, for example, sector i has a higher capital-labor ratio than sector h (i.e. $|\theta_{KL}^{ih}| > 0$), the increased demand for capital gives rise to a downward pressure on the output of sector i and the increased demand for labor gives rise to an upward pressure. This effect is represented by the second term.

It follows that if the direct (substitution) effect works strongly in the opposite direction to the indirect (cost-push) one, the rise in emission tax on an industry may raise the output of the industry.

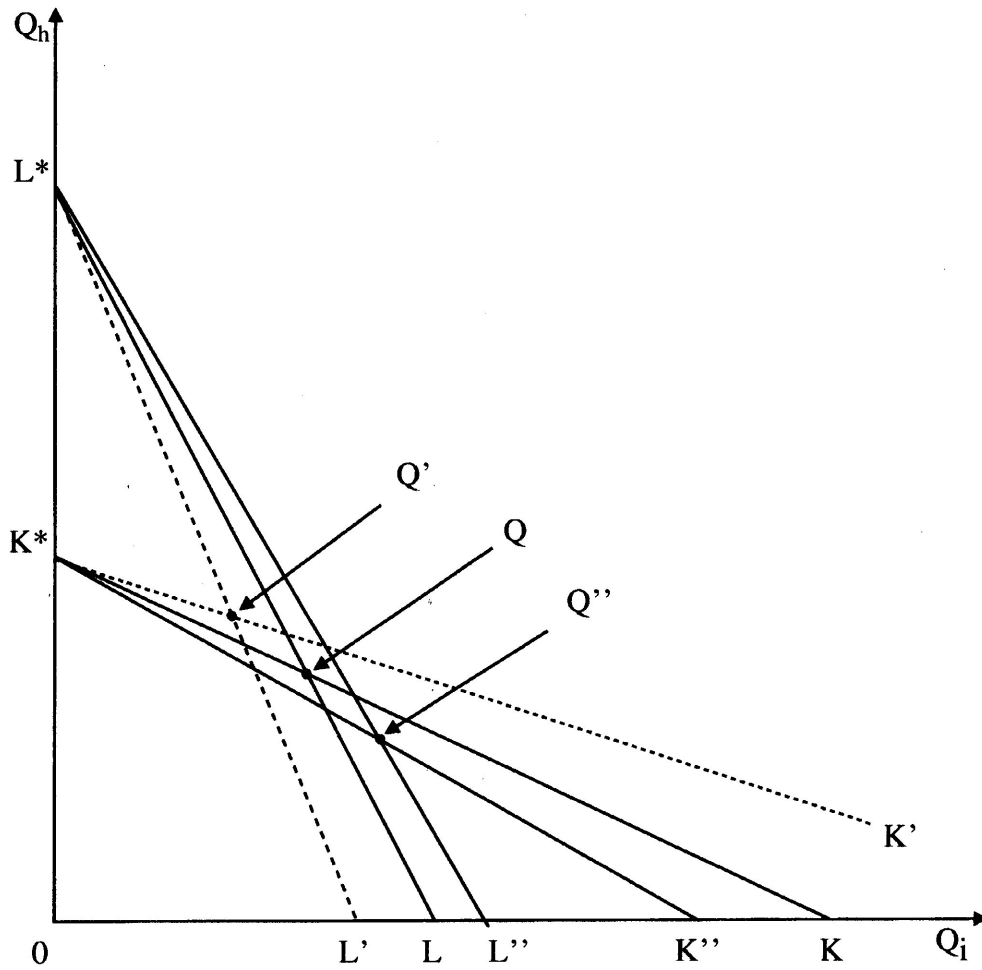


Figure 1:

Using Figure 1 and the numerical example above, let us explain two effects. Figure 1 depicts the equilibrium in the output space. Horizontal and vertical axes represent the

outputs of sector i and sector h respectively. Let the full employment lines for capital and labor at the initial equilibrium be denoted by line K^*K and L^*L whose slopes are given by a_{Ki}/a_{Kh} and a_{Li}/a_{Lh} . Since in the example, the capital-labor ratio in sector i is lower than that in sector h , line K^*K is flatter than line L^*L . The outputs at the initial equilibrium is given by the point Q where both factor markets are cleared.

Now suppose that emission tax on sector i is raised by 1%. First, let us consider the indirect effect. From (8) and (9), 1% rise in w_Z^i leads to 0.01/0.63% rise in w_K and 0.08/0.63% fall in w_L because the capital-labor ratio in sector i is lower than that in sector h . Since capital and labor are substitutes in sector i , these changes in factor prices lead to the fall in a_{Ki} and the rise in a_{Li} : $\hat{a}_{Ki}^{ID} = -0.06/0.63$ and $\hat{a}_{Li}^{ID} = 0.00645/0.63$ (the superscript ID means indirect effect). On the other hand, from the perfect complementarity between capital and labor in sector h , both \hat{a}_{Kh}^{ID} and \hat{a}_{Lh}^{ID} are zero. Therefore, by the indirect effect, the full employment line shift to K^*K' and L^*L' , and outputs shift to Q' where the output of sector 1 decreases. As already pointed out, the indirect effect always works in this direction.

Next, consider the direct effect. The direct effect of tax on input coefficients in sector i is given by $\hat{a}_{Ki}^D = \varepsilon_{KZ}^i = 1$ and $\hat{a}_{Li}^D = \varepsilon_{LZ}^i = -0.06$. Thus, the direct effect works in the opposite direction to the indirect one. Moreover, since the size of the direct effects is larger than that of the indirect effects (i.e. $|\hat{a}_{Ki}^D| > |\hat{a}_{Ki}^{ID}|$ and $|\hat{a}_{Li}^D| > |\hat{a}_{Li}^{ID}|$), the direct effects dominate the indirect one.

Taking into account two effects, the full employment lines shift to K^*K'' and L^*L'' and new equilibrium outputs shifts to Q'' . Therefore, in the example above, the rise in emission tax on sector i increases the output of that sector.

4 The effects of emission tax on the level of emission

Next, let us consider the effects of emission regulations on the level of emission.

From (4) and $\hat{v}_Z = \lambda_{Z1}\hat{v}_{Z1} + \lambda_{Z2}\hat{v}_{Z2}$, the rate of change in total emission is given by

$$\begin{aligned}\hat{v}_Z &= \sum_{i=1,2} \lambda_{Zi} \hat{Q}_i + \sum_{i=1,2} \lambda_{Zi} \hat{a}_{Zi} \\ &= \sum_{i=1,2} \lambda_{Zi} \hat{Q}_i + \sum_{j=K,L} \varepsilon_{Zj} \hat{w}_j + \sum_{i=1,2} \lambda_{Zi} \varepsilon_{ZZ}^i \hat{w}_Z^i\end{aligned}$$

By (8), (9), and (15), the above equation can be rewritten as

$$\hat{v}_Z = \frac{\beta}{|\theta_{KL}^{12} \lambda_{KL}^{12}|} (C^1 \hat{w}_Z^1 + C^2 \hat{w}_Z^2) \quad (16)$$

where

$$\begin{aligned}
C^i &= \theta_{Zi} [-(\theta_{Kh} + \theta_{Lh})\alpha_K(\theta_{Zi} - \theta_{Zh})\varepsilon_{KL} \\
&\quad + \theta_{Lh}\alpha_K(\theta_{Li} - \theta_{Lh})\varepsilon_{KZ} + \theta_{Kh}\alpha_L(\theta_{Ki} - \theta_{Kh})\varepsilon_{LZ}] \\
&\quad + \alpha^i |\theta_{KL}^{ih}| [\theta_{Ki}(\theta_{Li} - \theta_{Lh})\varepsilon_{KZ}^i - \theta_{Li}(\theta_{Ki} - \theta_{Kh})\varepsilon_{LZ}^i] \\
\beta &\equiv \alpha^1 \alpha^2 / \alpha_K \alpha_L \alpha_Z
\end{aligned}$$

First, we consider the effects of the uniform change in emission taxes, that is, we set $\hat{w}_Z^1 = \hat{w}_Z^2 = \hat{w}_Z$. Then, we can show that $\hat{v}_Z / \hat{w}_Z < 0$, that is, uniformly strengthening emission taxes on both industries always reduces the total level of emission (see Appendix 3).

Next, let us examine the sector specific change in emission tax. The sign of this effect is represented by C^i . Note that since C^i depends on both factor intensities and elasticities of substitution, the sign of C^i cannot be easily determined by some simple conditions. Therefore, we consider a special case in which $\varepsilon_{LZ}^i = 0$ for $i = 1, 2$. This means that labor and emission are neither substitutes nor complements in both sectors. In reality, this case seems plausible because in most realistic situations, emission is often more closely related to capital than labor.

In this case, C^i reduces to

$$\begin{aligned}
C^i &= \theta_{Zi} [-(\theta_{Kh} + \theta_{Lh})\alpha_K(\theta_{Zi} - \theta_{Zh})\varepsilon_{KL} + \theta_{Lh}\alpha_K(\theta_{Li} - \theta_{Lh})\varepsilon_{KZ}] \\
&\quad + \alpha^i |\theta_{KL}^{ih}| \theta_{Ki}(\theta_{Li} - \theta_{Lh})\varepsilon_{KZ}^i
\end{aligned}$$

This yields the following proposition.

Proposition 2 If $\varepsilon_{LZ}^1 = \varepsilon_{LZ}^2 = 0$, $a_{Ki}/a_{Kh} > a_{Li}/a_{Lh} > a_{Zi}/a_{Zh}$ and $\theta_{Li} > \theta_{Lh}$, \hat{v}_Z / \hat{w}_Z^i is positive.

Proof Since $\varepsilon_{LZ}^1 = \varepsilon_{LZ}^2 = 0$, we have $\varepsilon_{KL} > 0$, $\varepsilon_{KZ} > 0$, and $\varepsilon_{KZ}^i > 0$. And since $a_{Ki}/a_{Kh} > a_{Li}/a_{Lh} > a_{Zi}/a_{Zh}$, we have $\theta_{Ki} - \theta_{Kh} > 0$ and $\theta_{Zi} - \theta_{Zh} < 0$. Thus, the proposition immediately follows. Q.E.D.

This means that, for example, if sector 1 is relatively capital intensive and sector 2 is relatively emission intensive, and if the cost share of labor is larger in sector 1 than in sector 2, a rise in emission tax on sector 1 (holding emission tax on sector 2 constant) increases the total level of emission.

The intuitive reasoning is simple. Under the conditions in the proposition, the rise in emission tax on sector i decreases the output of sector i and increases that of sector h .

Since the expanded sector is relatively intensive in emission and the other sector is relatively non-intensive in emission, the rise in emission from the expanded sector dominates the fall in emission from the contracted sector, and thus, total emission increases.

The same kind of arguments is valid in the case of $\varepsilon_{KZ}^i = 0$. Therefore, we can conclude that strengthening emission tax may increase the total volume of emission according to the way in which taxes are imposed.

5 Concluding remarks

In this paper, we consider the two sector production economy with two primary factors and emission, and explore into the effects of emission tax. The following results are derived.

First, strengthening emission tax imposed on an industry may increase the outputs of the industry (proposition 1). Second, while strengthening emission taxes uniformly across industries always reduces emissions, strengthening emission tax unevenly may increase them (proposition 2). Although the international carbon-leakage attracts much attention, this paper shows that the carbon-leakage can occur *between industries within a country*.

The mechanism which brings about propositions 1 and 2 is the general equilibrium effect through the factor market adjustment, and the emission tax affected the factor demand through two effects (the cost-push and substitution effects), and note that both propositions cannot be derived in one primary factor economy like the one in Ishikawa and Kiyono (2000) because, as they show, when the policy instrument is emission tax, the production always specializes to one commodity in the one factor model.

Finally, we present some policy implications. As mentioned in the introduction, it is often believed that emission tax is the easier regulation to implement than permit trade in most cases. However, the results in this paper imply that emission tax may have unintended adverse effects on the level of emissions. Thus, emission permit trade is a more reliable policy instrument for emission restrictions than tax because it always can restrict emission to the intended level.

Appendix 1

Here, we show that when $\theta_{Ki} = \theta_{Lh} = 0.1$, $\theta_{Li} = \theta_{Kh} = 0.8$, $\theta_{Zi} = \theta_{Zh} = 0.1$, $\varepsilon_{KL}^i = \varepsilon_{KZ}^i = 1$, $\varepsilon_{LZ}^i = -0.06$ and when capital and labor are perfect complements in sector h , we have $\hat{Q}_i/\hat{w}_Z^i > 0$.

First, A_i^i can be rewritten as

$$A_i^i = -\frac{\alpha^i \alpha^h}{\alpha_K \alpha_L} X_i - \frac{(\alpha^h)^2}{\alpha_K \alpha_L} \theta_{Zi} \theta_{Kh} X_h$$

where

$$\begin{aligned} X_i &= (\theta_{Kh} + \theta_{Lh})^2 \theta_{Zi} \theta_{Ki} \varepsilon_{KL}^i + \theta_{Lh} \theta_{Ki} [\theta_{Lh} - \theta_{Li} (\theta_{Kh} + \theta_{Lh})] \varepsilon_{KZ}^i \\ &\quad + \theta_{Kh} \theta_{Li} [\theta_{Kh} - \theta_{Ki} (\theta_{Kh} + \theta_{Lh})] \varepsilon_{LZ}^i \\ X_h &= (\theta_{Kh} + \theta_{Lh})^2 \varepsilon_{KL}^h + \theta_{Lh}^2 \varepsilon_{KZ}^h + \theta_{Kh} \theta_{Lh} \varepsilon_{LZ}^h \end{aligned}$$

From the perfect complementarity between capital and labor in sector h and from (12), we have $\varepsilon_{KZ}^h = \varepsilon_{LZ}^h = -(1 + \theta_{Kh}/\theta_{Lh}) \varepsilon_{KL}^h$, thus $X_h = 0$.

On the other hand, inserting numerical values into X_i yields

$$\begin{aligned} X_i &= (0.8 + 0.1)^2 \times 0.1 \times 0.1 \times 1 + 0.1 \times 0.1 \times [0.1 - 0.8 \times (0.1 + 0.8)] \times 1 \\ &\quad + 0.8 \times 0.8 \times [0.8 - 0.1 \times (0.1 + 0.8)] \times (-0.06) \\ &= -0.02536 < 0 \end{aligned}$$

Therefore, we have $A_i^i > 0$. At last, the elasticity of substitution $\varepsilon_{LZ}^i = -0.06$ must satisfy the constraint (13). Since the RHS of (13) in the example is -0.0625 , this constraint is satisfied.

Appendix 2

The proof of $B_i^i \geq 0$. If all factors are substitutes, $B_i^i \geq 0$ is clear because we have $\varepsilon_{jl} > 0$ for $\forall j, l = K, L, Z$ $j \neq l$. Thus, we have to prove $B_i^i \geq 0$ when there is a pair of factors which are complements. We give the proof in the case of $\varepsilon_{KL}^i < 0$ and $\varepsilon_{KL}^h < 0$, that is, the case where capital and labor are complements in both sectors. The similar arguments can be applied to the other cases.

We can rewrite B_i^i as

$$\begin{aligned} B_i^i &= \frac{\alpha^h}{\alpha_K \alpha_L} \left\{ \alpha^i \left[(\theta_{Kh} + \theta_{Lh})^2 \theta_{Ki} \varepsilon_{KL}^i + \theta_{Lh}^2 \theta_{Ki} \varepsilon_{KZ}^i + \theta_{Kh}^2 \theta_{Li} \varepsilon_{LZ}^i \right] \right. \\ &\quad \left. + \alpha^h \left[(\theta_{Kh} + \theta_{Lh})^2 \theta_{Kh} \varepsilon_{KL}^h + \theta_{Lh}^2 \theta_{Kh} \varepsilon_{KZ}^h + \theta_{Kh}^2 \theta_{Lh} \varepsilon_{LZ}^h \right] \right\} \end{aligned}$$

Then, from (13), we have

$$\begin{aligned}
B_i^i &\geq \frac{\alpha^h}{\alpha_K \alpha_L} \left\{ \alpha^i \left[-(\theta_{Kh} + \theta_{Lh})^2 \theta_{Ki} \frac{\theta_{Li} \varepsilon_{KZ}^i \varepsilon_{LZ}^i}{\theta_{Ki} \varepsilon_{KZ}^i + \theta_{Li} \varepsilon_{LZ}^i} \right. \right. \\
&\quad \left. \left. + \theta_{Lh}^2 \theta_{Ki} \varepsilon_{KZ}^i + \theta_{Kh}^2 \theta_{Li} \varepsilon_{LZ}^i \right] \right. \\
&\quad \left. + \alpha^h \left[-(\theta_{Kh} + \theta_{Lh})^2 \theta_{Kh} \frac{\theta_{Lh} \varepsilon_{KZ}^h \varepsilon_{LZ}^h}{\theta_{Kh} \varepsilon_{KZ}^h + \theta_{Lh} \varepsilon_{LZ}^h} \right. \right. \\
&\quad \left. \left. + \theta_{Lh}^2 \theta_{Kh} \varepsilon_{KZ}^h + \theta_{Kh}^2 \theta_{Lh} \varepsilon_{LZ}^h \right] \right\} \\
&= \frac{\alpha^h}{\alpha_K \alpha_L} \left[\frac{\alpha^i}{\theta_{Ki} \varepsilon_{KZ}^i + \theta_{Li} \varepsilon_{LZ}^i} (\theta_{Ki} \theta_{Lh} \varepsilon_{KZ}^i - \theta_{Li} \theta_{Kh} \varepsilon_{LZ}^i)^2 \right. \\
&\quad \left. + \frac{\alpha^h}{\theta_{Kh} \varepsilon_{KZ}^h + \theta_{Lh} \varepsilon_{LZ}^h} (\theta_{Kh} \theta_{Lh})^2 (\varepsilon_{KZ}^h - \varepsilon_{LZ}^h)^2 \right] \\
&\geq 0
\end{aligned}$$

Appendix 3

The sign of \hat{v}_Z/\hat{w}_Z depends on the sign of $C^1 + C^2$. It is given by

$$\begin{aligned}
C_1 + C_2 &= -\beta[(\theta_{Z1} - \theta_{Z2})^2 \alpha_K \varepsilon_{KL} + (\theta_{L1} - \theta_{L2})^2 \alpha_K \varepsilon_{KZ} \\
&\quad + (\theta_{K1} - \theta_{K2})^2 \alpha_L \varepsilon_{LZ}]
\end{aligned}$$

Following the same procedure as when we prove $B_i^i \geq 0$, we can show $C^1 + C^2 \leq 0$. However there is an easier way to prove it.

Since the equivalence between tax and quota holds in the model, the effects of the uniform rise in emission tax can be derived from the effects of the reduction in country-wide emission quota. If we interpret emission quota as factor endowment, the latter effect has already been derived in Batra and Casas (1976), theorem 1: the decrease in the supply of a factor (emission quota) always raises the reward of that factor (emission permit price). This result and the equivalence between both policies imply that the uniform increase in emission tax always reduces the volume of emission.

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